Helicopter Vibration Reduction Using Structural Optimization with Aeroelastic/Multidisciplinary Constraints – A Survey

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This paper presents a survey of the state-of-the-art in the field of structural optimization when applied to vibration reduction of helicopters in forward flight with aeroelastic and multidisciplinary constraints. It emphasizes the application of the modern approach where the optimization is formulated as a mathematical programming problem, the objective function consists of the vibration levels at the hub, and behavior constraints are imposed on the blade frequencies and aeroelastic stability margins, as well as on a number of additional ingredients that can have a significant effect on the overall performance and flight mechanics of the helicopter. It is shown that the integrated multidisciplinary optimization of rotorcraft offers the potential for substantial improvements, which can be achieved by careful preliminary design and analysis without requiring additional hardware such as rotor vibration absorbers or isolation systems.

Nomenclature

b_s	= cross-sectional dimension, Fig. 2
b	= semichord
C_T	= thrust coefficient
C_W	= weight coefficient $= W/\pi\Omega^2 R^4$
D	= vector of design variables
e_1	= root offset of blade
EI_y, EI_z	= lag and flap stiffness, respectively
EI_{xxr}, EI_{zzr}	= flap and lag stiffness, respectively, at blade root
$F(\bar{D})$	= objective function, Eq. (12)
F_i	= generalized forcing function for ith blade mode
F_{XH}^{4P}, F_{YH}^{4P}, F_{ZH}^{4P}	= four per rev harmonic component of shear in hub plane (x and y direction) and vertical (z direction), respectively
GJ	= blade torsional stiffness
h_s	= cross-sectional dimension, Fig. 2
h_{sr}, h_{st}	= cross-sectional dimension at blade root and at tip, respectively
I_b	= flapping inertia of blade
I_i	= generalized inertia for ith blade mode
J	= mass polar moment of inertia of rotor
$J(D)$	= objective function
J_0	= initial value of mass polar moment of inertia of rotor
K_{FX}, K_{FY}, K_{FZ}, K_{MX}, K_{MY}, K_{MZ}, K_1, K_2, K_3, K_4	= weight factors equal to zero or one
l	= length of elastic part of blade
l_{0s}	= length defining station, where outboard blade segments start, Fig. 3
m	= mass per unit length of blade
m_{ns}	= nonstructural mass per unit length of blade
m_0	= reference mass per unit length
$M_{x1\max}$	= maximum peak-to-peak value of oscillatory hub rolling moment
M_{XH}^{4P}, M_{YH}^{4P}, M_{ZH}^{4P}	= four per rev harmonic component of pitching, rolling, and yawing moment, respectively
$P_{z1\max}$	= maximum value of oscillatory peak-to-peak vertical hub shear
R	= blade radius
t_h, t_b	= cross-sectional dimension, Fig. 2
t_{h1}, t_{h2}	= thickness of front and rear vertical wall of structural box
V_i	= vibration index for ith mode
W	= weight of helicopter
X_A	= offset from elastic axis to cross-sectional aerodynamic center
X_I	= offset from elastic axis to cross-sectional center of mass
X_m	= offset from counterweight to elastic axis
y_{ns}	= chordwise offset of nonstructural mass from elastic axis
y_0	= chordwise offset of c.g. from elastic axis, baseline configuration
α_k	= real part of characteristic exponent, Eq. (14)
β_p	= precone
ζ_i	= viscous modal damping in ith mode
ζ_k	= real part of eigenvalue in hover
Λ	= sweep angle
λ	= taper ratio of blade
μ	= advance ratio
σ	= blade solidity
σ_{allow}	= allowable normal stress
τ_{allow}	= allowable shear stress
ϕ_i	= mode shape for ith mode, Eq. (12)
Ω	= rotor angular velocity
ω	= forcing frequency in vibration index, Eq. (18), or natural frequency
ω_i	= natural frequency of ith mode
ω_k	= imaginary part of eigenvalue
ω_U, ω_L	= upper and lower bounds on natural frequency, respectively

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I. Introduction and Historical Perspective

VIBRATION levels encountered in helicopters have been one of the most important criteria for distinguishing between successful and less successful designs. During the last two decades, at least two major helicopter production contracts have been awarded on the basis of criteria associated with vibration levels. During the same period, criteria for vibration levels at typical locations in the fuselage, such as the pilot seat, have become much more stringent. For example, 15 years ago vibration levels in the neighborhood of 0.10 g were considered acceptable, in modern helicopters 0.05 g levels are desirable, and the requirements for the next generation of helicopters will probably be in the 0.02–0.05 g range. It is evident from two excellent reviews dealing with a wide array of vibration problems and their control in rotorcraft, written by Reichert[1] and Loewy[2], that such problems have been frequently treated, by passive or active means, after they have manifested themselves in the flight test phase.

Since the principal source of vibrations is the main rotor, it has been recognized a long time ago that reducing vibrations at their source is very cost effective. Attempts were made in the mid-1950s to reduce vibration levels in the rotor by a chordwise and spanwise placement of concentrated masses, denoted tuning masses, and to reduce vibration levels in rotor blades. References 3–6 are representative of such early attempts. Much later, as the art of rotor blade design became more sophisticated, aerodynamic means such as blade twist and use of swept tips on rotor blades were identified in an excellent paper by Blackwell[7] as a practical means for controlling vibrations at their source, i.e., the rotor.

Helicopter vibrations are generated by a combination of aerodynamic, dynamic, and structural sources that form the basis of any aeroelastic response calculation. However, it should be emphasized that for a helicopter these three groups represent only a restricted set of disciplines when one seeks the best possible design.

Among all flying vehicles the helicopter is inherently interdisciplinary since the flexibility of the blades couples with the aerodynamics and dynamics so as to influence vehicle performance, flight mechanics, and handling qualities as well as acoustics. Furthermore, in the next generations of helicopters, the disciplines cited above will also couple with the control system, which can and probably will be used for stability augmentation, vibration reduction, and suppression of potential aeromechanical instabilities. Finally, it should be noted that the vibration problem is further complicated by the coupling between the rotor and fuselage, which determines the precise manner in which the vibratory loads from the rotor excite the fuselage.

The optimum design of a helicopter for minimum vibrations in forward flight clearly represents a complex structural optimization problem with multidisciplinary constraints. Thus, the simultaneous consideration of the disciplines mentioned is essential for the successful design of helicopters. Ideally, the best designs will be obtained only if vibration levels are considered early in the design process before the numerous design variables are fixed due to the conflicting considerations introduced by the interdisciplinary nature of the problem. Thus, automated structural design techniques have to be applied early in the preliminary design process when the largest number of design variables can be determined based on the simultaneous application of multidisciplinary constraints. This is the only approach that can guarantee substantial benefits and produce a vibration-free, optimally designed helicopter.

The field of structural optimization, or structural synthesis, has become a practical tool in recent years as a result of three decades of extensive research and development.[8-10] This is due to significant advances in computers coupled with a large body of research, which has led to more efficient methods using a combination of mathematical programming techniques for optimization, approximation concepts[11] for improved efficiency, and the finite element method for structural modeling. Structural optimization has found considerable use in the aerospace industry; however, until recently most of the applications have been for fixed-wing aircraft.[12,13] Application of formal structural optimization to helicopter problems started in the early 1980s, and it generated considerable interest in industry, academia, and research organizations. A detailed review of the application of numerical optimization to helicopter design problems, including structural optimization, can be found in a paper written by Miura.[14] The various techniques available for vibration reduction in rotorcraft using structural optimization were explored and reviewed in considerable detail in Ref. 15.

It is interesting to note that while the helicopter community started using structural optimization techniques at least 10 years after such studies emerged in the fixed-wing area, the community has been quick to recognize the remarkable payoffs available, and presently the use of structural optimization, with interdisciplinary constraints, for these two types of vehicles is almost on an equal footing. The primary reason for the rapid acceptance of structural optimization in the rotary-wing field is due to both the inherently interdisciplinary nature of the problem and the fact that the high potential payoff for vibration reduction of helicopters in forward flight, using structural optimization, has a central role in rotorcraft and is less important in fixed-wing aircraft.

The main objective of this paper is to present the current state-of-the-art in the field of structural optimization when applied to vibration reduction of helicopters in forward flight with aeroelastic and multidisciplinary constraints. For convenience, the research described here is separated into two groups. The first group consists of research carried out at universities; the second group presents the research performed in industry and government organizations. This body of research enables one to draw useful conclusions on the important ingredients that need to be incorporated in current and future research. Furthermore, it also enables one to make useful recommendations on the future developments that are desirable for the design of the next generation of vibration-free rotorcraft.

II. Research Performed at Universities

Most of the research done at universities during the last nine years has been aimed at the design of blade configurations which would have low vibration characteristics in forward flight. A description of this research in chronological order is presented next.

The first documented application of optimum structural design to the vibration reduction problem of a helicopter in forward flight while simultaneously enforcing aeroelastic constraints was presented in Refs. 16–18. A concise description of this research is provided below.

The helicopter vibration reduction problem is expressed as a general class of structural synthesis problems consisting of preassigned blade properties and helicopter performance parameters, design variables, objective functions, behavior constraints, and side constraints. This optimum design problem, which is solved using mathematical programming methods, is stated in the following mathematical form.

Find the vector of design variables $\boldsymbol{D}$ such that

$$g_q(\boldsymbol{D}) \geq 0, \qquad q = 1,\ldots,Q \tag{1}$$

$$D_i^{(L)} \leq D_i \leq D_i^{(U)}, \qquad i = 1,\ldots,n_{\mathrm{dv}} \tag{2}$$

$$\text{and } J(\boldsymbol{D}) \to \min \tag{3}$$

where $g_q(\boldsymbol{D})$ is the qth constraint function in terms of the design variables $\boldsymbol{D}$; D_i is the ith design variable, superscripts L and U denote lower and upper bounds, respectively; and $J(\boldsymbol{D})$ is the objective function in terms of the design variables.

The system considered in these studies[16-18] was a four-bladed hingeless rotor, attached to a rigid fuselage. The fuselage degrees of freedom were not included, and the complicated mechanism of vibration transfer to a flexible fuselage was also excluded. This idealized configuration is shown in Fig. 1. Quantities defining the helicopter blade configuration such as b, β_p, e_1, and x_A were treated as preassigned parameters. Helicopter performance parameters that define the helicopter flight condition in trimmed flight are the advance ratio μ and the weight coefficient C_W. These were assumed to be specified parameters describing the configuration.

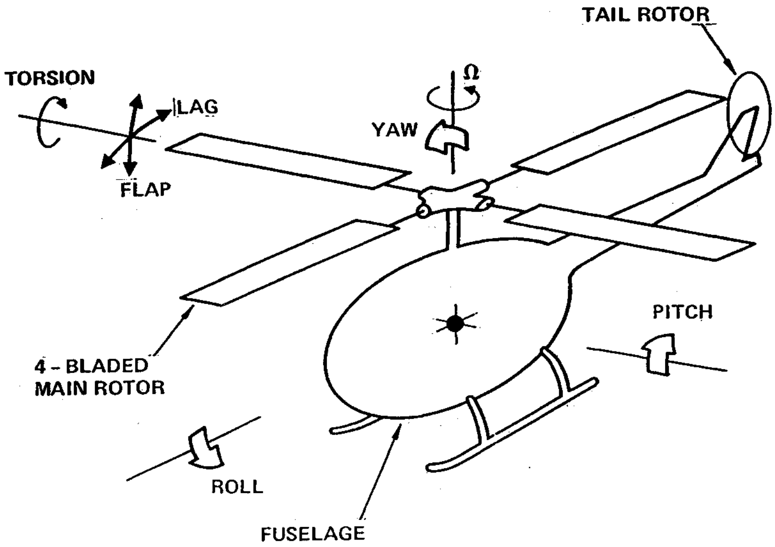

Fig. 1 Helicopter rotor fuselage model.

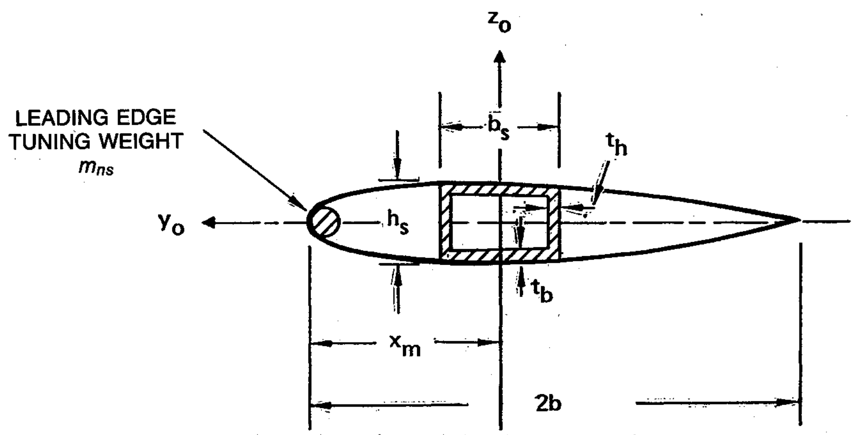

Fig. 2 Typical blade cross section.

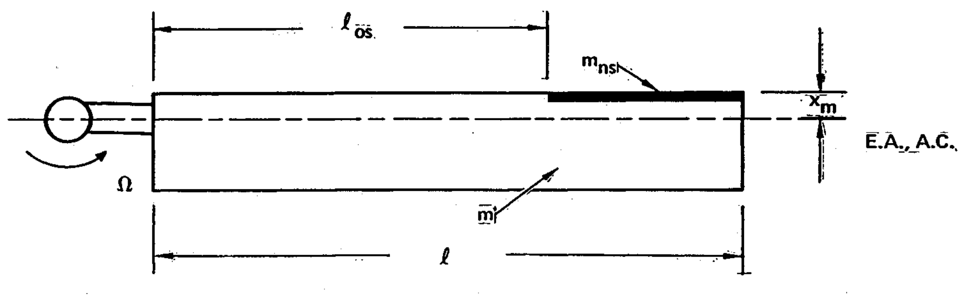

Fig. 3 Spanwise and chordwise location of nonstructural mass (the figure shows the leading-edge location; when $x_m = 0$, mass is on elastic axis; E. A. and A. C. denote the elastic axis and aerodynamic center, respectively.

The *design variables* are associated with a typical cross section of the rotor blade shown in Fig. 2. For each cross section, the design variables are the breadth b_s, the height h_s, and the thicknesses t_b and t_h of the thin-walled, rectangular box section representing the structural member at each spanwise station. A total of seven spanwise stations are used for a finite element analysis[19] of the blade which produces the free vibration mode shapes and frequencies of the rotating blade. Elastic properties of the blade in bending and torsion as well as the structural mass properties are expressed in terms of these design variables. The nonstructural mass of the blade is assumed to consist of two parts. The first part is the nonstructural skin and honeycomb case surrounding the structural cell shown in Fig. 2, which provides the appropriate aerodynamic shape and is assumed to be a fixed percentage of the critical blade mass. The second contribution to nonstructural mass is represented by m_{ns} in Fig. 2, which is a counterweight used as a tuning device for controlling blade frequency placement and the section center of gravity location. The nonstructural masses m_{ns} at three outboard stations of the blade, see Fig. 3, corresponding to the two outboard finite elements, are also used as design variables, and the offsets x_m from the elastic axis are given parameters.

Two types of *behavior constraints* are used. The frequency constraints are expressed in terms of nondimensional rotating frequencies ω^2 of the blade in the flap, lag, and torsional degrees of freedom. These uncoupled rotating frequencies are obtained from a Galerkin-type finite element analysis.[19] The fundamental frequencies in flap, lag, and torsion are required to fall between preassigned upper and lower bounds. The frequency placement constraint is expressed mathematically in the form:

$$g(D) = \frac{\omega^2}{\omega_U^2} - 1 \leq 0 \tag{4}$$

$$g(D) = 1 - \frac{\omega^2}{\omega_L^2} \leq 0 \tag{5}$$

Equations (4) and (5) are written for each of the three fundamental frequencies of the blade in flap, lag, and torsion, providing a total of six behavior constraints. The higher frequencies are constrained to avoid four per rev (4/rev) resonances in the four-bladed hingeless rotor considered in this study.

The second type of frequency constraints are placed on aeroelastic stability margins. The aeroelastic constraints represent the requirement that stability margins in hover remain virtually unchanged during the optimization process. For soft-in-plane blade configurations, (almost all hingeless rotors in production belong to this category), the effect of forward flight is usually stabilizing,[20] therefore, this assumption is quite reasonable. The aeroelastic stability constraints are expressed mathematically in the form

$$g(D) = \frac{\zeta_k}{\zeta_k^{(L)}} - 1 \geq 0, \qquad k = 1,2,...,6 \tag{6}$$

For hover, the aeroelastic analysis, based on six modes,[20] produces eigenvalues that occur in complex conjugate pairs

$$\lambda_k = \zeta_k \pm i\omega_k \tag{7}$$

The blade is stable when $\zeta_k < 0$, for $k = 1, 2,...,6$. In Eq. (6), $\zeta_k^{(L)}$ represents a lower bound on ζ_k, which was selected such that aeroelastic stability margins in hover are not reduced as a result of an optimization process. *Side constraints* were also placed on the design variables t_h, t_b, b_s, h_s, and m_{ns} (see Figs. 2 and 3) in the form of upper and lower bounds to prevent the variables from reaching impractical values during the optimization process.

The *objective function* to be minimized was a mathematical expression representing the maximum peak-to-peak value of the oscillatory vertical hub shears or the oscillatory hub rolling moments, at an advanced ratio of $\mu = 0.30$:

$$\left.\begin{aligned} J(\bar{D}) &= P_{z|\max} \\ \text{or} \quad J(\bar{D}) &= M_{x|\max} \end{aligned}\right\} \tag{8}$$

These objective functions are obtained by using the steady-state blade response values in flap, lag, and torsion generated by the aeroelastic stability and response analysis of a trimmed rotor blade in forward flight.[20] The trim is limited to flap trim, and it is not coupled to the aeroelastic analysis. A brief description of the relations between the aeroelastic analysis, the loads acting on the blade, and the hub shears and moments can be found in Ref. 16; complete details can be found in Ref. 18.

The optimization problem described above was solved using a general-purpose constrained optimization package NEWSUMT developed by Miura and Schmit.[21] It is based on an extended interior penalty function and Newton's method with approximate second derivatives.[22] Furthermore, approximation concepts were used in the optimization process to reduce costs.[11,23] The organization of the optimization process used in Refs. 16–18 is illustrated in Fig. 4 and is briefly described below.

1) An initial trial design D_0 is chosen by selecting the values of b_s, h_s, t_h, t_b at the seven spanwise stations and m_{ms} at the three outboard stations.

2) The uncoupled rotating modes and frequencies of the blade are obtained using a finite element model. Explicit first- and second-order Taylor series approximations to the frequency constraints are calculated in closed form.

3) The aeroelastic stability in hover, the response in forward flight, and the vertical hub shears and moment (which constitute the objective function to be minimized) are calculated.[20] The gradient information for the explicit approximation of the objective function and aeroelastic constraints is calculated by finite differences.

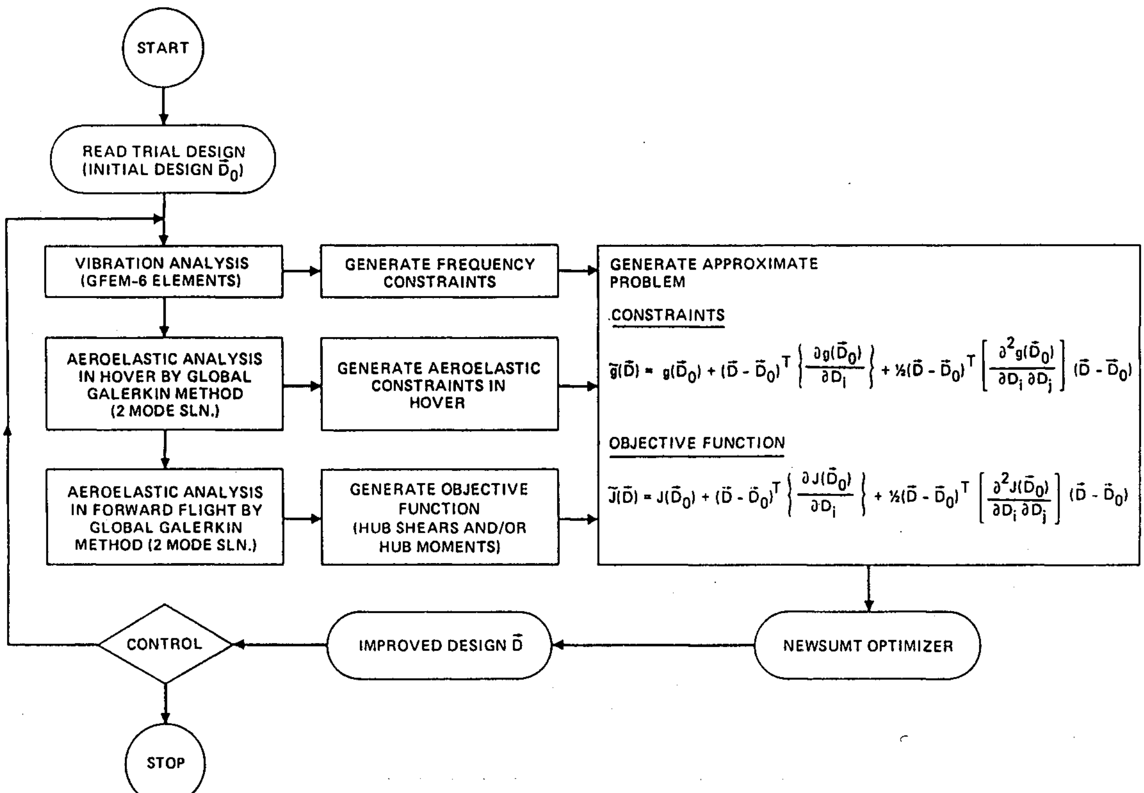

Fig. 4 Basic organization of the optimization process.

4) The mathematical programming problem represented by Eqs. (1–3) is replaced by an approximate problem where the constraints $g_q(D)$ and the objective function $J(D)$ are expressed by Taylor series approximations. The approximate problem is solved by the NEWSUMT optimizer to obtain an improved design.

5) The entire optimization process is repeated with the improved design as starting point until the sequence of vectors D converges to a solution D^* where all inequality constraints are satisfied and $J(D^*)$ is at least a local minimum.

It should be noted that due to the excessive computer time involved, the optimization procedure described above was not exercised in a completely automated manner. It was used in an interactive manner, with manual interruptions so as to reduce computing costs associated with generating the substantial amount of numerical results presented in Refs. 16–18. These results have indicated that by applying modern structural optimization to the design of soft-in-plane hingeless rotors, vibratory hub shears in forward flight can be reduced by 15–40%. The reduction is achieved by relatively small modifications of the original design, which yielded optimal frequency placement in fundamental flap, lag, and torsion modes. These percentages represent conservative estimates since the original design was a soft-in-plane blade resembling an MBB 105 hingeless blade, which is known to be a good design. The best

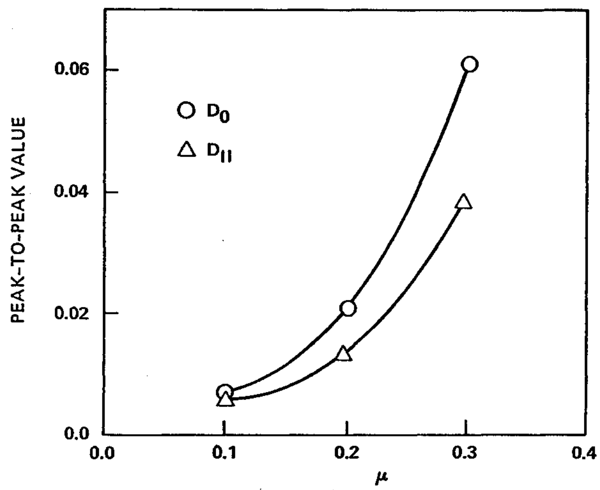

Fig. 5 Vertical hub shears, nonlinear, peak-to-peak values, nondimensionalized ($P_{Z1}l/\Omega^2 I_b$) vs μ, comparison of initial and final designs after two stages of optimization, (D_0-initial design, D_{II}-design after two stages of optimization).

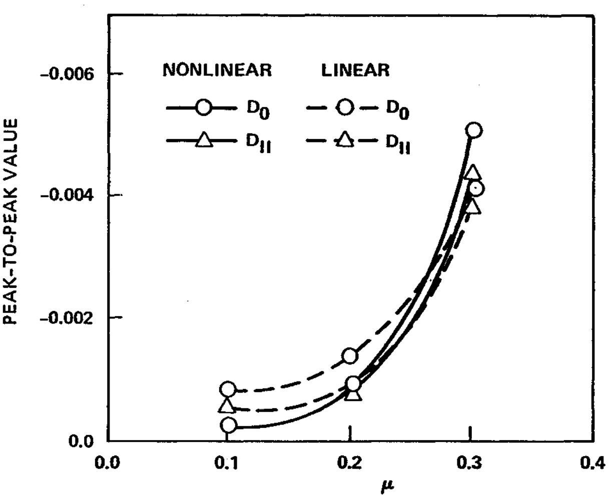

Fig. 6 In-plane hub shears, peak-to-peak values, nondimensionalized ($P_{Y1}l/\Omega^2 I_b$) vs μ, comparison on initial and final designs after two stages of optimization, (D_0-initial design, D_{II}-design after two stages of optimization).

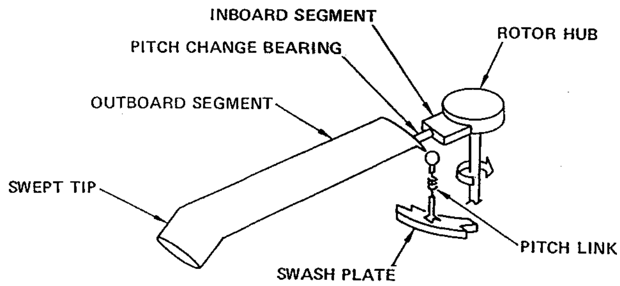

Fig. 7 Swept tip hingeless rotor blade model.

results were obtained when using nonstructural mass, as additional design variables, in the outboard segment of the blade and placed on the elastic axis. An interesting byproduct of optimization were final blade configurations, which were 9–20% lighter than the initial uniform blade. This result was obtained without using blade weight as the objective function in the optimization process. Finally, it should be noted that use of the linear oscillatory hub shears at an advance ratio of $\mu = 0.30$ represents as acceptable the objective function for the optimum design of soft-in-plane hingeless rotor blades.

From the numerous results presented in Refs. 16–18, two figures are worthwhile to include here because they have some basic features that were also noted in other studies reviewed here. The nonlinear vertical hub shears over the advance ratio range of $0<\mu<0.30$ are shown in Fig. 5, indicating a consistent reduction in the hub shears over the whole range of advance ratios. These results demonstrate that the choice of the linear vertical hub shear at a moderately high advance ratio ($\mu = 0.30$) as the objective function yielded reductions in the oscillatory vertical hub shears at intermediate advance ratios. In Fig. 5, D_0 denotes the initial design and D_{II} denotes the final design after two stages of optimization. For design, D_{II}, the linear peak-to-peak vertical hub shear, was reduced by 37.9% (nonlinear hub shear contains the geometrically nonlinear terms due to moderate blade deflections), and the nonlinear hub shear was reduced by 35.9%. The corresponding reduction in hub rolling moments, which were not used as an objective function, was 24.17% and 25.2%, respectively. The blade nonstructural mass added for this case in the form of tuning weights was 2.3% of blade weight. A byproduct of the optimization was a 19.7% reduction in total blade mass. Figure 6 illustrates the behavior of linear and nonlinear peak-to-peak values of the in-plane shear. The reduction of hub shears for design D_{II} compared to D_0 is evident at $\mu = 0.30$, but the reduction is small and nonuniform for the range $0.1<\mu<0.30$. This reflects on the well-known sensitivity of in-plane hub shears to higher-order nonlinear terms, associated with the lag degree of freedom.

Further studies by Celi and Friedmann have considered the structural optimization problem, with aeroelastic constraints, of rotor blades with straight and swept tips.[24,25] The configurations considered in Refs. 24 and 25 were soft-in-plane, four-bladed, hingeless rotors with straight and swept tips. The basic blade configuration is shown in Fig. 7; the improved model can represent both single cell and double cell cross sections as illustrated in Fig. 8.

In Refs. 24 and 25, mathematical statement of the optimization problem consists again of Eqs. (1–3). The general formulation,[25] which also includes tuning masses, can accommodate up to nine design variables for each cross section. However, in the actual optimization studies undertaken, the number of cross-sectional design variables was limited to two. For both single cell and double cell cross sections, the independent design variables consisted of t_1 and x_2 shown in Fig. 8. The single cell case of $x_2/h_s = 4.5$ and the double cell case $x_1 = x_2/2$ were kept constant. The outside wall of the double cell has the shape of a NACA 0012 airfoil. The blade tip sweep angle Λ was also used as a design variable.

The behavior constraints used were similar to the constraints used in the earlier work. For frequency constraints, Eqs. (4) and (5) were used again. Two types of aeroelastic stability constraints were used. Again, it was assumed that for soft-in-plane blade configuration, maintaining aeroelastic stability margins for hover would be adequate. The more stringent aeroelastic stability constraint used was

$$g(D) = 1 - \frac{\zeta_k}{0.95\zeta_{kB}} \leq 0, \qquad k = 1,...,6 \tag{9}$$

which expresses the requirement that the loss of stability in a given mode should not exceed 5% of the baseline value ζ_{kB}. A more relaxed aeroelastic stability constraint

$$g(D) = \zeta_k \leq 0, \qquad k = 1,2,...,6 \tag{10}$$

which was also used, represents the requirement that the blade be stable in hover. This more relaxed stability constraint was found to be ineffective in the numerical studies.

In addition to the behavior constraints on frequency placement and aeroelastic stability margins, a new constraint on autorotation was also introduced. The autorotation constraint is expressed mathematically as

$$g(D) = 1 - \frac{J}{0.9J_0} \leq 0 \tag{11}$$

This constraint, Eq. (11), expresses the requirement that the mass polar moment of inertia of the rotor J retain at least 90% of its initial value J_0 during optimization.

The objective function used in this study was the peak-to-peak value of the 4/rev vertical hub shears, corresponding to the first part of Eq. (8); for the swept tip case, it was found that normalizing it by the thrust coefficient, i.e., $P_{z1\max}/C_W$, is useful since it compensates for the deficiencies associated with the trim procedure.

The optimization problem described so far resembles the previous formulation.[16-18] Therefore, it is important to mention the main differences between this study[24] and the previous study.[16-18] From the structural optimization point of view, the main differences are associated with the construction of the approximate problem and the solution of the optimization problem. In this study it was recognized that the most expensive function to evaluate is the objective function. An effective method for building an approximation to the objective function is to use a Taylor Series approximation in terms of the design variables[11]

$$F(D) \approx F(D_0) + \nabla F(D_0)\delta D + \tfrac{1}{2}\delta D^T\left[H(D_0)\right]\delta D \tag{12}$$

where $F(D)$ is the objective function, D_0 is the current design, and $\nabla F(D_0)$ and $[H(D_0)]$ are respectively the gradient and the Hessian matrix at the current design. The Hessian matrix is the matrix of the second partial derivatives of the objection function with respect to the design variables. The vector δD is defined as

$$\delta D = D - D_0 \tag{13}$$

In Refs. 16–18, finite differences were used for the construction of the derivative information in Eq. (12). This approach led to excessive computing costs. In this study an alternative approximation technique, introduced by Vanderplaats,[26,27] was used. This alternative technique is based on the idea of approximating the gradient the Hessian in Eq. (12), by using design information available at the time. The details of the implementation of this technique for the hub shear minimization problem can be found in Refs. 24 and 25.

The approximate constrained optimization problem was solved using the feasible direction code CONMIN.[28] The aeroelastic analysis from which the vibratory hub loads and aeroelastic constraints were obtained is described in Refs. 29 and 30. The trim procedure used was similar to that used in the previous study.[16-18] With the exceptions noted here, the basic organization of the optimization process was similar to that described in Fig. 4.

The optimization results obtained[24,25] indicated vibration reductions between 20–50%. The additional vibration reduction obtained from using the sweep angle as an additional design variable was on the order of 10%. These results can be considered to be conservative, since tuning masses were not used as design variables in the optimization. The method used for constructing the approximate problem (objective function) was found to be quite efficient. However, it was felt that better approximations could be obtained by using analytical gradients.

The most recent and to some comprehensive study of vibration reduction in helicopter rotor blades with aeroelastic constraints has been carried out by Lim and Chopra.[31,32] Whereas this study resembles the previous studies cited, it also contains a number of new and substantial contributions. The optimum design problem is represented again by Eqs. (1–3). The configuration considered is a four-bladed soft-in-plane hingeless rotor attached to a fuselage of infinite mass, and the preassigned parameters resemble those used in the previous studies.[16-18,24,25] The configuration of the airfoil, including a tuning mass and the representation of the blade by finite elements, is illustrated in Fig. 9.

The design variables at each blade cross section are m_{ns}, y_0, EI_y, EI_z, and GJ. The blade is represented by five finite elements, leading to a total of 30 design variables. The linkage between the stiffness properties and the actual cross-sectional dimensions is not made in this study.

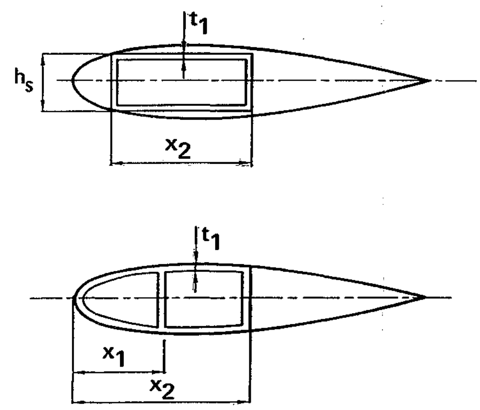

Fig. 8 Single cell and double cell blade cross sections.

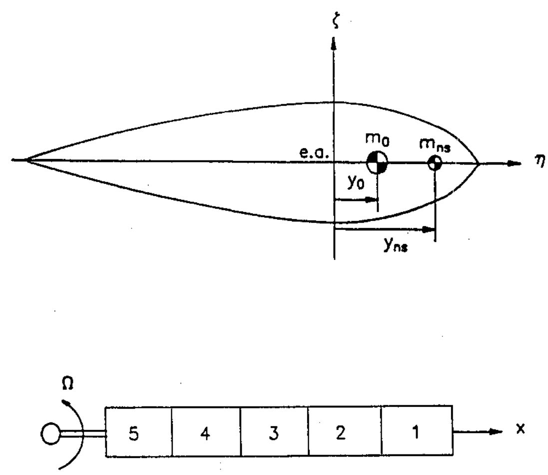

Fig. 9 Configuration of the airfoil including a tuning mass and representation of blade finite elements.

The behavior constraints consist of the requirement that the fundamental flap, lag, and torsional mode of the blade be stable in forward flight. Mathematically, this constraint is expressed as

$$g(D) = \alpha_k \leq 0, \qquad k = 1,2,3 \tag{14}$$

where α_k is the real part of the characteristic exponent associated with three fundamental modes. Although this constraint appears to be more accurate than the hover constraint used in previous studies,[16-18,24,25] for the case of a soft-in-plane hingeless blade, it may actually be a somewhat weak constraint. First, because the soft-in-plane blades are most unstable in hover,[20] imposing such a constraint in forward flight without enforcing a similar constraint in hover can be dangerous. Next, requiring that the blade be stable is not sufficient; a blade should have a predetermined aeroelastic stability margin that should not change substantially during the optimization process.

Furthermore, it should be noted that frequency placement constraints and autorotation constraints, imposed in Refs. 24 and 25, are not used in this study. Indeed, the authors impose the side constraints on blade and mass stiffness properties, which the authors imply will produce favorable frequency placement.

Two possible alternatives are considered for the objective function. One is the sum of the 4/rev harmonics of the hub loads in the hub nonrotating frame:

$$J(D) = K_{FX}F_{XH}^{4P} + K_{FY}F_{YH}^{4P} + K_{FZ}F_{ZH}^{4P} + K_{MX}M_{XH}^{4P} + K_{MY}M_{YH}^{4P} + K_{MZ}M_{ZH}^{4P} \tag{15}$$

where the hub forces and moments are nondimensionalized by $m_0\Omega^2R^2$ and $m_0\Omega^2R^3$, respectively. A second objective function is defined as the sum of the hub force resultant and moment resultant in the hub-fixed nonrotating frame:

$$J(D) = K_F\sqrt{(F_{XH}^{4P})^2 + (F_{YH}^{4P})^2 + (F_{ZH}^{4P})^2} + K_M\sqrt{(M_{XH}^{4P})^2 + (M_{YH}^{4P})^2 + (M_{ZH}^{4P})^2} \tag{16}$$

Both objective functions are evaluated for an advance ratio of $\mu = 0.30$, as in Refs. 16–18, 24, and 25. The optimization problem is solved using the CONMIN code.[28] Each optimization iteration involves updating the search direction from the sensitivity analysis and determining the optimum move parameter by polynomial approximation in the one-dimensional search.

An important contribution made in Refs. 31 and 32 consists of the formulation of a direct analytical approach for the calculation of the derivatives of the hub loads and blade stability with respect to the design variables. A detailed description of the sensitivity analysis of the vibratory hub loads, using a finite element formulation in space and time, is described in Refs. 32 and 33. A similar sensitivity analysis for the aeroelastic stability of the blade as represented by the real part of the characteristic exponent was presented in Refs. 32 and 34. These sensitivity derivatives are obtained at a fraction of the computational cost associated with the more conventional finite difference method used in Refs. 16–18. Although the benefits of using sensitivity analyses in structural optimization are well known,[22,35] this is the first time that such an analysis has been developed for complicated problems such as hub loads and blade stability in forward flight. Another contribution in the analysis used for this optimization study was incorporation of a trim procedure which is fully coupled with the aeroelastic response analysis.[31,32] This represents a substantial improvement on the trim procedures used in the previous studies,[16-18,24,25] where the propulsive trim procedure was not fully coupled with the aeroelastic response analysis.

The optimization studies conducted[31,32] showed that use of Eq. (16) with $K_F = K_M = 1$ is preferable to using Eq. (15) with $K_{FZ} = 1$ and $K_{FX} = K_{FY} = K_{MX} = K_{MY} = K_{MZ} = 0$. Next, a wide range of combinations of cross-sectional design variables was explored. It was found that a combination of cross-sectional design variables consisting of m_{ns}, y_{ns}, EI_y, EI_Z, and GJ represents the most effective combination, since it takes advantage of both distribution on nonstructural masses and their location as well as modifications of blade stiffnesses. The results obtained after eight iterations, with 25 design variables, are shown in Fig. 10. It is evident from Fig. 10 that a 77% reduction of the objective function was achieved, which represents the largest reduction obtained in this study. For the various other cases considered, reductions in the objective function in the range of 20–50% were observed.

The most important conclusions obtained in these studies were 1) analytical sensitivity derivatives for hub loads and aeroelastic constraints produce large reductions in computer times and should be used in future studies, and 2) the use of combined objective functions such as Eq. (16) is preferable to minimizing vertical hub shear alone.

In addition to the fairly comprehensive studies described above, there were a number of less ambitious studies conducted at a number of universities, where the main objective was to explore some fundamental aspects of the structural optimization problem. A representative study in this category, Ref. 36, was aimed at application of structural optimization to rotor blade frequency placement. This study is based on the premise that separation of blade flap, lead-lag, and torsion frequencies from the aerodynamic forcing frequencies, which occur at integer multiples of the rotor revolutions per minute will also reduce vibration in forward flight. The optimum structural design problem is one in which the mass and stiffness distributions are selected by an optimization process, such that the uncoupled flap, lag, and torsional rotating frequencies are placed in certain predetermined "windows," which are separated from the integer resonances. Aeroelastic stability constraints were not included in Ref. 26, and direct minimization of hub loads resulting from the aeroelastic response of the blade was not incorporated in this study.

Again the optimization problem is cast in the form represented by Eqs. (1–3). The cross section of the blade model was similar to that shown in Fig. 2, except that the nonstructural mass at each cross section consisted of a tuning mass m_{ns}, shown in Fig. 2, together with a second lumped mass M_L inside the structural box (this mass is not shown in Fig. 2). The design variables were the dimensions b_s, h_s and the thicknesses as t_b and t_h of the thin-walled rectangular box section, representing the structural member, at each spanwise station. The vibration analysis, from which mode shapes and frequencies are obtained, is based on a tapered finite element model, using 10 spanwise stations.[36]

The behavior constraints were "frequency windows" on the first and second flapwise bending mode frequencies, expressed as

$$\omega_{1f}^{(L)} < \omega_{1F} < \omega_{1F}^{(U)}$$

$$\omega_{2F}^{(L)} < \omega_{2F} < \omega_{2F}^{(U)}$$

These constraints are recast in the form of Eqs. (4) and (5). Constraints were also imposed on rotor inertia for autorotation, in a form similar to Eq. (11). A third constraint on the maximum stresses due to centrifugal loads were also imposed. Side constraints were placed on the design variables to prevent them from reaching impractical values. A constraint was also placed on the physical dimensions of the structural cell so that it would fit within the airfoil.

Two objective functions were used: 1) minimize discrepancies between desired and actual frequencies, and 2) minimize weight. The optimization problem was solved using CONMIN.[28] The actual helicopter rotor blade optimization studies were somewhat limited in scope. First, an articulated rotor blade was optimized, and a weight reduction of 26% was obtained. Next, a teetering rotor blade was optimized, without any significant reduction in blade weight.

Peters and Cheng[37] have continued the research on the blade frequency placement problem.[36] A blade resembling the AH-64A blade geometry was investigated, and the principal emphasis was on satisfying stress constraints, assuming that the harmonic blade loads are obtained from a flight loads survey. The results obtained indicated that stress constraints based on fatigue life considerations can be added to the frequency placement problem. However, the authors concluded that the feasibility of a completely integrated (i.e., including additional disciplines such as aerodynamic performance) and totally automated design process is still in the distant future.

A very interesting study, which has a truly multidisciplinary flavor, was recently completed by Celi.[38] In this study the torsional stiffness of a hingeless rotor blade and its cross-sectional offsets are designed so as to stablize the phugoid oscillation of the helicopter by increasing the stablizing effect of the rotor. The optimization problem is again cast in the forms of Eqs. (1-3). The objective function is the real part of a complex eigenvalue which characterizes the phugoid mode, at $\mu = 0.30$. The design variables are blade torsional stiffness GJ and blade cross-sectional offsets x_A and x_I. The behavior constraints are 1) aeroelastic stability constraints in forward flight, same as Eq. (14); 2) blade root load constraints, in the rotating system, expressing the requirement that the maximum peak-to-peak flap bending moment and root torsional moment should not increase by more than 10% from their baseline values; and 3) a cyclic pitch response constraint representing a handling quality type of constraint. The objective function and the behavior constraints are approximated by Eqs. (12) and (13), using the approach described before.[24,25] The optimization problem is solved using CONMIN. The results obtained indicated that approach is efficient, and phugoid modes can be stabilized. Although this study was not aimed at vibration reduction, it nevertheless illustrates how handling quality type of constraints can be incorporated in vibration reduction studies.

III. Research Carried Out in Industry and Government Research Organizations

The research described in this section is divided into a number of parts, each associated with a particular organization where the research was carried out. An attempt was made to describe this research in a chronological order.

A. Research at United Technologies Research Center

United Technologies Research Center has traditionally had a longstanding interest in rotor blade design with dynamic constraints. In 1971, Bielawa,[39] in a pioneering study, considered the minimum weight design problem of a helicopter rotor blade in hover. Behavior constraints were imposed on bending-torsion (flap-pitch) aeroelastic stability as well as on the frequency placement. The five design variables included the width and depth of the spar at the blade tip, chordwise position of the spar center, chordwise dimension of the counterweight (tuning mass, Fig. 2), and its chordwise position (at 30% span and 70% span). A linear aeroelastic analysis was used and analytical expressions for the flutter eigenvalue sensitivity were obtained. The optimization algorithm was a Lagrange multiplier method. The weight reduction achieved fluctuated between 3.5-9% of the initial weight of the blade. The important contribution of this study was the recognition of the importance of structural synthesis in blade design, almost a decade before the topic became popular, and also the first use of a sensitivity analysis for this class of problems. However, this study did not address the vibration reduction problem directly.

Subsequently, Taylor[40] considered the vibration reduction problem in rotor blades in forward flight using modal shap-

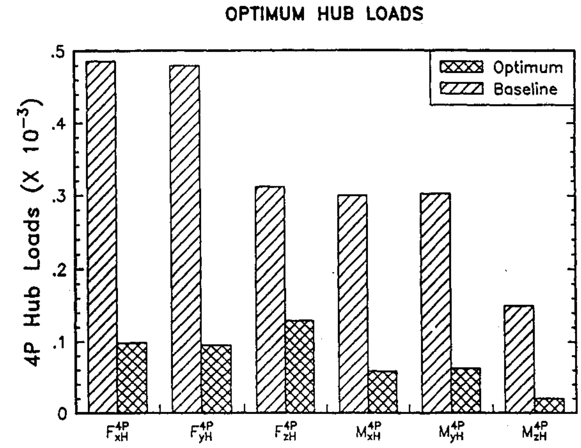

Fig. 10 Optimum 4/rev hub forces and moments for $\mu = 0.30$, $C_T/\sigma = 0.07$; $N_b = 4$ (Ref. 31); hub forces and moments are nondimensionalized with resect to $m_0\Omega^2R^2$ and $m_0\Omega^2R^3$, respectively.

ing. This approach is based on the idea of reducing vibration levels by modifying the mass distribution and, to a lesser extent, the stiffness distribution of the blade, in order to reduce a so-called "model shaping parameter" (MSP). The MSP was based on the flapwise modes, and it was not a function of the mode natural frequency or the forcing frequency. However, MSP could be used to reduce the sensitivity of the blade modes to selected harmonics of the air load and thus can be interpreted as an ad hoc type of optimality criterion. Strictly speaking, Taylor's study is not an optimum design approach; however, MSP can be easily incorporated in a blade structural optimization study by using it as an objective or constraint function.

Building upon this foundation, some of the most important contributions to blade design optimization have been made more recently by Davis and Weller.[41-43] In the first study,[41] four different problems representative of a hierarchy of optimization problems, from relatively simple to complex, were considered. The analytical studies dealt with four specific problems, namely: 1) maximization of bearingless rotor structural damping; 2) blade natural frequency placement; 3) minimization of hub modal shears; and 4) minimization of modal vibration indices. Only the last three topics, which are within the scope of this survey will be discussed.

The three problems are cast in the standard mathematical programming form represented by Eqs. (1-3). The optimization problem is solved using the automated design synthesis (ADS) package,[44] and various options of the program were exercised to determine which optimization algorithm provided the best performance. The coupled free vibration modal analysis of the rotating blades was based on a Holzer-Myklestad method.[41] The most important aspects of these three structural optimization problems are briefly reviewed below.

Blade Natural Frequency Placement

The design variables used were EI_y, EI_z, and blade mass, at 11 spanwise sections resulting in 33 design variables. For an articulated rotor, the first three elastic flapwise modes and the second chordwise mode were placed. A constraint of $\pm 5\%$ was placed on total blade weight and rotor inertia changes. A multicomponent objective function

$$J(D) = \sqrt{k_1(\omega_1 - \omega)^2 + k_2(\omega_2 - \omega)^2 + k_3(\omega_3 - \omega)^2 k_4(\omega_4 - \omega)^2} \tag{17}$$

was used to produce frequency placement. Other alternatives to Eq. (17) were also explored; however, overall this objective function appeared to be the most effective.

Minimization of Hub Modal Shears

The design variables and constraints were similar to the previous problem. Modal hub shears for three critical modes of an articulated rotor, namely second and third flap and second chordwise, were minimized.

Minimization of Modal Vibration Indices

The design variables, initial design, frequency placement, and constraints were similar to the previous two problems. The objective function was vibration index given by

$$V_i = \frac{\left[\omega_i^2 \int_0^l \phi_i \mathrm{d}m\right] F_i}{\omega_i^2 I_i \sqrt{[1-(\omega/\omega_i)^2]^2 + [2\zeta_i(\omega/\omega_i)]^2}} \tag{18}$$

It should be noted that Eq. (18) represents an improved version of this expression presented originally by Blackwell.[7] It also represents an improvement on the MSP parameter presented initially in Ref. 40, which did not contain the frequency nor the amplication factor represented by the square root in Eq. (18). Although the MSP was utilized only for flapping modes,[40] it could have been used for the other modes, i.e., lead-lag and torsion, as well. The three critical modes were the same as for the previous problem. Five indices are used for each critical mode,[41] leading to a total of 15 objective function components; the optimization study led to a 40–70% reduction in the vibration indices.

Finally, the three approaches—frequency placement, hub shear minimization, and vibration index minimization—were ranked based on their ability to reduce vibrations. Frequency placement was found to be least effective. The use of hub shears in the objective function resulted in a significant reduction in the sum of the squares of all of the 15 vibration indices, compared to the frequency placement solution. Direct minimization of all of the vibration indices was the best, but not by much.

Using this study[41] as a starting point, Weller and Davis[42,43] carried out a monumental study in which 19 blade model configurations, including three baseline designs obtained using the design methods given in Ref. 41, were wind tunnel tested. Vibratory responses of four-bladed articulated model rotors were compared to assess the effectiveness of using modal shear reduction, modal vibration index reduction, and frequency placement to improve rotor vibratory response. The experimental setup, dynamically scaled rotor model description, and other relevant details are given in Refs. 42 and 43. Each blade could simulate changes in spanwise mass distribution as well as stiffness distribution in the flapwise and chordwise directions. Furthermore, mass and stiffness could be varied independently of each other. During the tests, the first harmonic blade flap was trimmed out and 4P fixed system shear and overturning moment (pitch and roll) were generated from the rotating measurement.

Three baseline configurations (soft, nominal, stiff) were used. Using the analytical optimization capability[41] described previously, the baseline configurations were redesigned for reduced vibrations using three approaches: 1) minimizing modal shears; 2) minimizing modal vibration indices; and 3) placing natural frequencies. Tests on the baseline and opti-

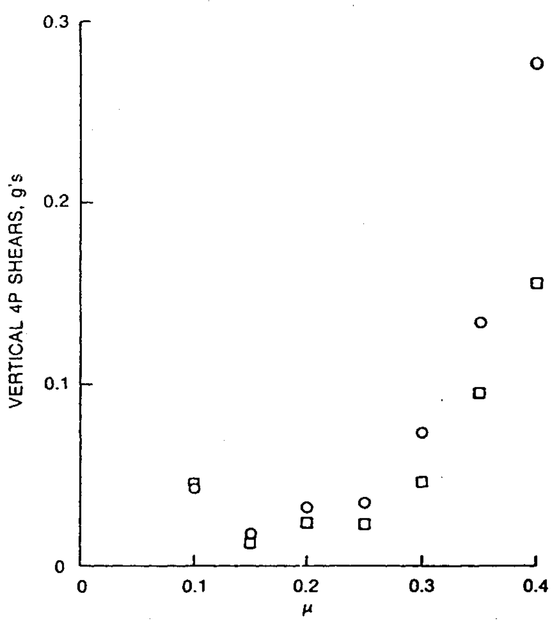

Fig. 11 Effect on vertical vibratory shear of minimizing modal shears by mass variation, nominal blade, 1 *g* flight condition, shears nondimensionalized by 1 *g* thrust[42]; circles denote baseline configuration, squares denote optimized configuration.

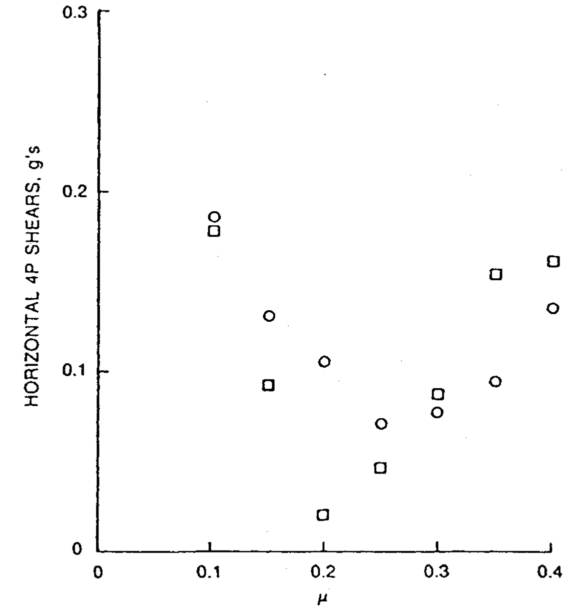

Fig. 12 Effect on horizontal vibratory shear of minimizing modal shears by mass variation, nominal blade, 1 *g* flight condition, shears nondimenstionalized by 1 *g* thrust[42]; circles denote baseline configuration, squares denote optimized configuration.

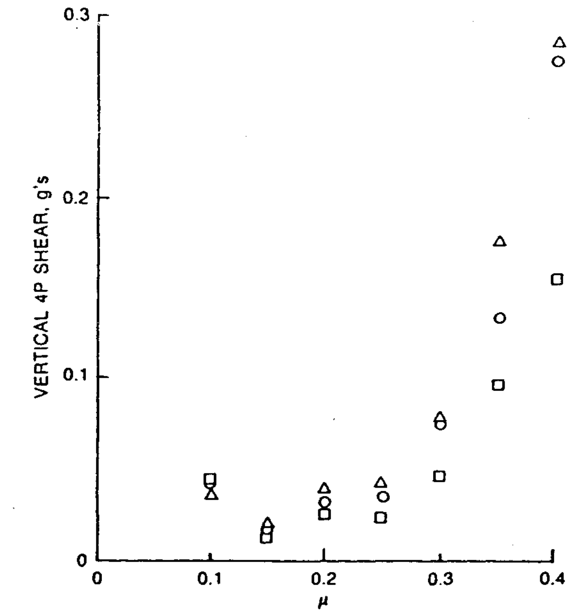

Fig. 13 Comparison of vertical vibratory shear for two configurations with matched natural frequencies, 1 *g* flight condition, shears normalized by 1 *g* thrust[42]; circles denote baseline configuration, squares denote optimized configuration using minimum modal shears, triangles denote optimized configuration using frequency placement alone.

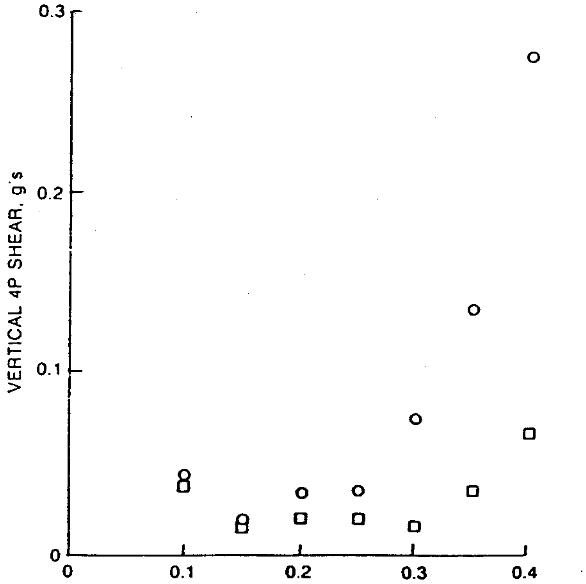

Fig. 14 Attenuation of vibratory response by structural tailoring of the blade design, 1 *g* flight condition, shears normalized by 1 *g* thrust[42]; circles denote baseline design, squares denote optimized design.

mized configurations were carried out to determine the relative merits of these three optimality criteria. Numerous results[42,43] were presented, and a few selected results are reproduced here since they demonstrate in an unequivocal manner certain fundamental aspects of the rotor blade structural optimization problem for reduced vibration levels.

Figure 11 depicts the change in measure 4P vertical shear between the baseline nominal configuration and the optimum design, when modal shears are minimized by mass variation only. The largest vibration level reductions (30–44%) are evident at the higher range of advance ratios $0.30<\mu<0.40$. Figure 12 shows a comparison of the in-plane hub shears for the same case. This figure raises a number of interesting questions, because it should exhibit a trend similar to Fig. 6. Comparing Figs. 5 and 6, it is evident that in-plane shears are approximately an order of magnitude smaller than vertical shears, and strangely this is not the case when one compares Figs. 11 and 12. It is also evident from Fig. 12 that the modified rotor exhibits a lower 4P in-plane shear response at low-to-moderate advance ratios whereas at high advance ratios, the modified rotor exhibits higher in-plane vibration levels. The authors note that although this behavior resembles Fig. 6, the evidence is not conclusive because the in-plane impedance of the wind-tunnel model was not sufficiently high to represent the locked-hub condition for which the analysis was performed.

Figure 13 compares measured 4P vertical shear for two rotors that have the same predicted frequency placement for the second and third flapwise and the second chordwise modes. The circles in Fig. 13 denote the baseline nominal configuration. The squares represent a configuration corresponding to a mass distribution where modal shears are minimized. The triangles denote a configuration that was designed to match the frequencies of the optimal configuration (squares) without minimizing the modal shears. Clearly, the vertical vibrations for the matched frequency blades are significantly higher than those for the blade designed to minimize vibratory response, even though their frequencies are matched. These results imply that optimization studies based on frequency placement do not guarantee vibration reduction in forward flight.

Figure 14 compares the attenuation of vibratory response by structural tailoring of the blade design involving both changes in the mass distribution as well as the stiffness of the blade. The circles in Fig. 14 represent the baseline nominal configuration. The squares represent a stiff blade with a mass distribution obtained from minimizing the vibration indices for three critical modes (second and third flap, second chordwise). The reduction in vibration levels at the higher range of advance ratios is remarkable; approximately 75% reduction in 4P shear is obtained.

The most important conclusions demonstrated in these studies[42,43] are briefly summarized. It was demonstrated that the modal-based optimization analysis can produce blade design with substantially lower vibration levels. The greatest levels of vibration reduction were observed at the higher advance ratios, $0.3<\mu<0.40$. While mass variations alone were adequate to significantly reduce vibration levels, compared to the baseline configuration, combined change of both mass and stiffness produce the best results. Frequency placement without regard to hub shear or vibration index values was shown to be inadequate to achieve minimum vibration levels. The calculated vibration indices were, in general, conservative and underpredicted the actual vibration reduction levels measured at the highest advance ratios. This is quite remarkable since vibration indices are single-valued and cannot reflect a vibration in air loading or response sensitivity due to changes in operating conditions.

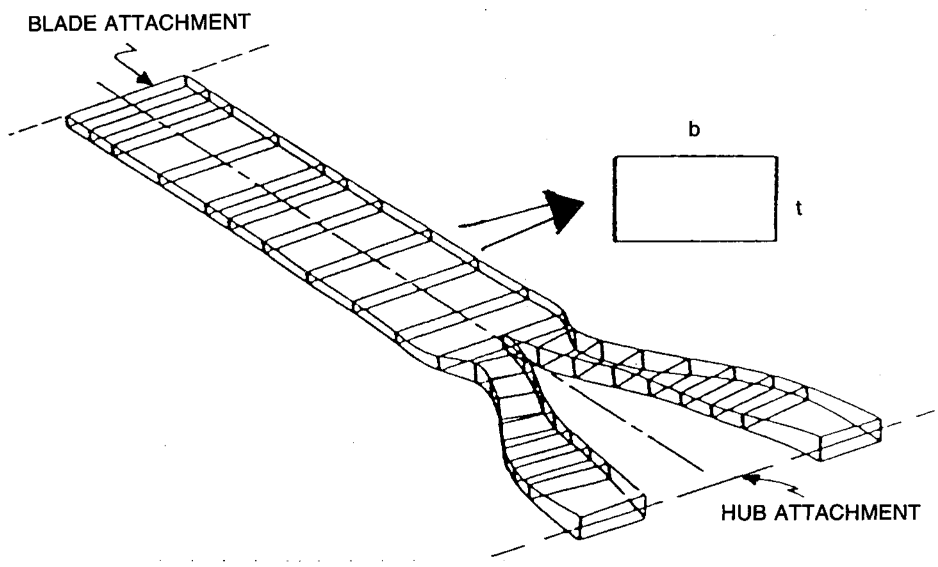

Fig. 15 Flexbeam configuration with a rectangular cross section.[48]

B. Research at Bell Helicopter Company

The application of formal mathematical programming to helicopter design problems was initiated at Bell in the early 1980s by Bennett.[45] Among the various examples considered,[45] a simple problem representing minimization of vertical hub shears due to blade flapping was treated using formal optimization. The design variables were the mass and stiffness properties of the blade at 15 radial stations. The objective function consisted of a mode shape parameter, somewhat similar to the modal shaping parameter developed by Taylor.[40] A Myklestad program was used to calculate the coupled natural frequencies and mode shapes of the blade. Equality constraints, requiring that blade weight and flapping inertia remain fixed during the optimization process, were imposed. The optimization problem was solved using the OPT program. As a result of the optimization, vertical shear was reduced by 60%.

Additional research on this topic was described by Yen.[46] He showed that significant reduction in oscillatory hub loads could be achieved by structural optimization of rotor blades. Yen emphasized the important role of the interaction between rotor structural properties and unsteady aerodynamic loads for minimizing hub loads. The analysis used simplified, linear rotor dynamic equations. A discussion of limited correlation studies among theory, wind-tunnel, and flight tests was also presented.

C. Research at McDonnell Douglas Helicopter Company

The application of numerical optimization methods to helicopter design problems at the McDonnell Douglas Helicopter Company (MDHC) is described in a recent paper by Banerjee and Shanthakumaran.[47] Structural optimization has been applied to three separate problems: 1) aeroelastic tailoring of rotor blades; 2) structural optimization of the fuselage; and 3) optimal design of advanced composite rotating flexbeams.[48] A brief description of these topics is provided next.

The studies on aeroelastic tailoring of rotor blades[47] were based on combining a Rotor/Airframe Comprehensive Aeroelastic Program (RACAP) with an optimizer such as ADS or CONMIN. The RACAP program is based on a coupled flap-lag-torsional model of the blades using a transfer matrix approach.[49,50] The rotor combined via a six degree of freedom complex hub impedance model to a NASTRAN finite element model of the fuselage. An iterative trim, air loads, and aeroelastic approach is used to obtain a response solution. The optimization problem is formulated using Eqs. (1-3). The design variables are EI_y, EI_z, GJ, and the mass m along the blade. Side constraints imposed on blade properties are described in Ref. 47. The behavior constraints considered were upper and lower bounds on blade moment of inertia to satisfy autorotation requirements. The objective function minimized was the blade response. Frequency constraints were used only in a selective manner. The constraints and the objective functions were approximated using second-order Taylor series expansion, Eq. (12), and the gradient information was generated by finite differences. Surprisingly low, 5-10%, reductions in vibration levels were obtained.

The structural optimization of the fuselage was based on a detailed NASTRAN model, with considerable design variable linkage.[47] The weight was minimized while requiring acceleration levels below desired values at specific locations in the fuselage. The lowest fuselage modes were also constrained to be within specified frequency bounds. In addition, the dynamic displacements at selected locations and the element dynamic stresses were constrained to be below specified upper bound values.

A detailed study of optimal design of an advanced/composite flexbeam for bearingless rotor applications is presented in Ref. 48. The flexbeam configuration optimized is shown in Fig. 15. The optimization is accomplished by combining a finite element analysis model for a multiple load path cantilevered beam under centrifugal loads called HUBFLEX with the ADS optimizer. The design variables are b and t, shown in Fig. 15, for each element of the flexbeam. The constraints are stress constraints

$$\sigma_{yi}/\sigma_{\text{allow}} \leq 1.0; \qquad \tau_{yi}/\tau_{\text{allow}} \leq 1$$

where σ_y is the normal stress and τ_y is the shear stress. The objective function was to minimize the peak value of the combined stresses along the flexbeam for conditions of limit, maximum fatigue, and endurance loads. An alternative objective was to maximize the first in-plane damping represented by damper motion per degree lag while constraining stresses within allowable limits. Side constraints were also imposed on cross-sectional dimensions and effective virtual flap and lag hinge offsets. The principal accomplishment of this research was to reduce the time required to design an optimal configuration from six months to one week, as a result of the automated design procedure.

D. Other Industrial Research

A few additional studies are briefly mentioned here for the sake of completeness. These represent preliminary attempts to apply structural optimization to problems that fall within the scope of this survey. Reference 51 represents an early attempt to design the flexbeam of a bearingless rotor so as to minimize combinations of bending and axial stresses for a given oscillatory load. In Ref. 52, Perley describes the use of regression analysis as a design optimization tool; among the various applications, the vibration reduction problem is also briefly discussed.

E. Research at NASA Langley Research Center

A very substantial research activity in the field of structural optimization of rotor blades has been nurtured under the auspices of the Interdisciplinary Research Office and the Aerostructures Directorate USAARTA-AVSCOM at NASA Langley Research Center. It should be emphasized that this activity is quite broad and extends beyond the topics discussed in this paper. The most important facets of this research activity, which are aimed at vibration reduction in helicopters, can be found in Refs. 53-55.

In Ref. 53 the authors address the problem of finding the optimum combination of tuning masses on a rotor blade so as to minimize blade root vertical shear, while avoiding excessive mass penalty. The design variables in this study are the tuning masses and their spanwise locations along the elastic axis of an idealized beam, which is assumed to represent the blade. The blade is represented by a simple finite element model of a rotating beam and only flapping motion is considered. Frequency constraints similar to Eqs. (4) and (5) are imposed on the first two flapwise modes. Three different objective functions are developed and tested. The optimization problem was solved using CONMIN. The first objective function minimizes the MSP associated with the two modes, multiplied by the total sum of the tuning masses. The second objective function reduces the shears, multiplied by the total sum of the tuning masses. The third minimizes the shear as a function of time, multipled by the sum of the tuning masses. Sensitivity derivatives are obtained analytically for the performance parameters and constraints to facilitate optimization. The first objective function was found to be ineffective when dealing with more than one mode. The other two objective functions proved to be effective for the multiple mode and multiple load case and produced excellent shear reduction with a low mass penalty. This study represents a useful fundamental contribution to optimizing rotating beams under prescribed dynamic loading conditions; however, it is yet to be applied to represent a practical helicopter vibration problem.

Reference 54 describes the minimum weight design of helicopter rotor blades with constraints on flap and lag frequencies; autorotation and stresses due to centrifugal loads were considered. This study represents essentially an improvement on Ref. 35 and is based on a similar blade model. The typical cross section is similar to Fig. 2, except that the tuning mass is replaced by a lumped mass inside the structural box, on the line of shear centers (elastic axis), and the thickness of the vertical wall of the structural box t_h (see Fig. 2) can have a value t_{h1} in the front and t_{h2} at the rear (or trailing edge). The structural box can also have linear taper, and for this case, the height at the root h_{sr} and the taper ratio $\lambda = h_{sr}/h_{st}$ are additional design variables. The objective function is a combination of structural weight and the nonstructural weight. The autorotation constraint is similar to Eq. (11), the frequency constraints are similar to Eqs. (4) and (5). Frequency constraints are placed on five frequencies associated with first two elastic flap and the first three elastic lag modes. The rotor is assumed to consist of four articulated blades. The blade is assumed to be torsionally rigid; the torsional stiffness is represented by a root spring, simulating control system stiffness. The stress constraints consist simply of the requirement that the stress due to centrifugal forces not exceed a given value in any segment of the structural box representing the blade. The blade was divided into 10 segments, and the mode shapes and frequencies were calculated using the free vibration module of the CAMRAD code.[56] First-order Taylor series are used to generate approximations to the objective function and the constraints. A sensitivity analysis is used to obtain analytical derivatives of the objective function, the autorotational, and the stress constraints. A central finite difference scheme is used to generate the derivatives of the frequency constraints. The CONMIN program is used to solve the optimization problem.

Optimization results for both a rectangular rotor and tapered rotor blade were obtained. For each blade three separate cases were considered: 1) all five frequency constraints, autorotation and stress constraints are imposed; 2) all five frequency constraints and autorotation constraints are used; and 3) frequency constraints on two flap modes only and autorotational constraints are used. The weight savings for cases 1 through 3 were 2.67, 3.04, and 8.50%, respectively, for the uniform blade. Somewhat higher numbers were obtained for the tapered blade. These results indicate that as additional constraints are imposed, the weight savings become fairly modest.

A much more ambitious extension of this research is presented in Ref. 55, which describes a study where integrated aerodynamic/dynamic optimization was performed so that blade weight and 4/rev vertical hub shear in forward flight were minimized simultaneously for a four-bladed articulated rotor. The rotor blade model, shown in Fig. 16, represents a modified Black Hawk blade developed for wind-tunnel testing. The design variables are EI_{xxr}, EI_{zzr}, λ, c_r, k_r, and w_j at seven spanwise stations. Three different objective functions are used: 1) blade weight consisting of the sum of structural and nonstructural weight; 2) 4/rev vertical shear F_z (at an advance ratio $\mu = 0.30$); and 3) weight and 4/rev vertical shear are minimized simultaneously using a global criteria approach.[55] The behavior constraints imposed are similar to those in the previous study,[54] except that only constraints on the first four flexible frequencies are introduced. The analysis is based on the CAMRAD code,[56] which is used to trim the

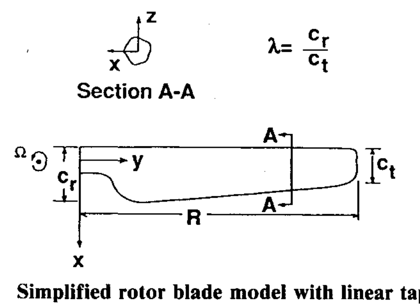

Fig. 16 Simplified rotor blade model with linear taper.[55]

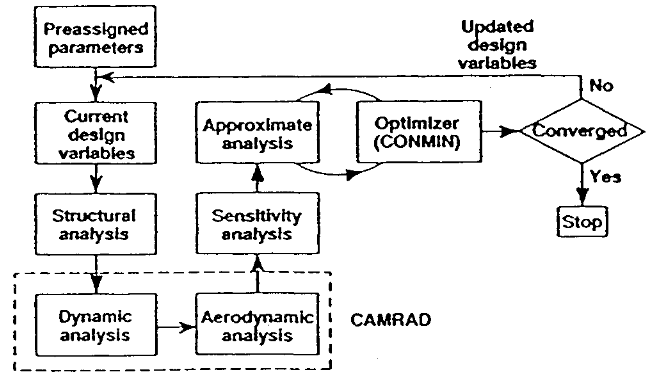

Fig. 17 Basic organization of the optimization procedure.[55]

rotor, calculate the 4/rev shears, and compute the mode shapes and frequencies. The approximation for the objective function and constraints is similar to the previous study.[54] The gradient information for hub shears is obtained by finite differences. The basic organization of the optimization procedure is shown in Fig. 17. The optimization problem is solved using the CONMIN code.

Three sample optimization test cases were used for this model rotor. For the first case, the weight alone is used as an objective function. This leads to a 7.5% reduction in blade weight and reduction of 40.6% in 4/rev vertical shear. In the second case, the 4/rev hub shear alone is minimized, which leads to a 85.6% reduction in hub shear, accompanied by an 8.5% reduction in blade weight. For the third case, the objective function represents a simultaneous minimization of blade weight and hub shear using the global criteria approach. For this case a 77.6% reduction in hub shears and a 10.6% reduction in blade weight is obtained. This paper contains numerous results together with a careful interpretation of the results. It is also shown that the power requirement of the rotor is reduced as a byproduct of optimization.

IV. Concluding Remarks

Based on the detailed survey presented in this paper, one can assess the state-of-the-art in the field of vibration reduction in helicopters using structural optimization with aeroelastic and multidisciplinary constraints. One can also try to predict some of the future developments in this field.

Although activity in this field started at least 10 years after similar endeavors were initiated in the fixed-wing area, the level of activity devoted to rotary-wing vehicles has advanced rapidly, and the level of sophistication evident in recent studies is comparable to the fixed-wing field. In one particular area, i.e., experiment verification of optimization design based on wind-tunnel testing of dynamically scaled model rotors,[42,43] the rotor-wing research has surpassed the work done on fixed-wing vehicles, since comparable tests have not been reported in the literature. This research[42,43] has also clearly demonstrated that frequency placement alone is not an effective method for reducing vibration levels in forward fight.

It is evident that useful vibration reduction studies in forward flight need to include the hub shears and, for hingeless rotors, even the hub overturning moments in the objective function. Behavior constraints need to be imposed on frequency placement, rotor inertia for autorotation, stresses due to centrifugal effects, and aeroelastic stability margins. Studies that have included most of these constraints and are representative of the current state-of-the-art include Refs. 16-18, 24, 25, 31, 32, 41, 43, and 55. The range of 4/rev hub shear reduction, achieved in four-bladed rotors, evident from these studies varies between 20-75%. The higher end of the range (75%) can be achieved only by using nonstructural tuning masses in addition to modifications in the mass and stiffness distribution of the blade. Thus, use of such tuning masses plays an important role in this class of problems. Furthermore, in a number of studies, weight reductions on the order of 7-17% were obtained as a result of optimization, and it is interesting to note that such weight reductions were obtained even without blade weight as the objective function.

In most studies where 4/rev hub shears were used in the objective function, the hub shears were calculated at a particular value of the advance ratio, usually $\mu = 0.30$. This approach, first used in Ref. 16, was adopted in other studies,[24,25,31,32,47,55] and it produces fairly uniform hub shear reduction in the whole range of advance ratios for straight blades.

Two approaches for minimizing vibration levels have emerged. The first approach is based on actually calculating the 4/rev hub shears from an aeroelastic response analysis,[16-18,24,25,47,55] including a trim calculation. This approach, which produces uniform reduction in hub shear throughout the whole range of advance ratios, is analytically more complicated and computationally much more expensive than the second approach. It also significantly increases the complexity of the structural optimization process. The second alternative is based on using modal shaping parameters, modal shear, or vibration indices.[41-43,45,53] Among these the vibration index used in Refs. 41-43 appears to be promising. This second approach reduces the complexity of the calculations, since an aeroelastic response calculation is circumvented. However, it is important to note that the vibration indices are single-valued and cannot reflect a variation in air loading or response sensitivity due to changes in operation conditions. This is perhaps the reason that the actual measured vibration reductions in Refs. 42 and 43 exceed the predicted values, which were usually found to be conservative. It is possible that calculation of the vibration indices was based on an assumed air load distribution corresponding to a fairly high advance ratio; therefore, the more impressive vibration reductions, shown in Figs. 11, 13, and 14, were evident mainly at high advance ratios, $0.30 < \mu < 0.40$. When hub shears are based on a complete aeroelastic response analysis, such as Fig. 5, a uniform hub shear reduction is observed in the whole advance ratio range. This also implies that using hub shears or carefully selected vibration indices as an objective function produces a robust optimum design for the blade, which is not very sensitive to off-design operating conditions.

Due to the significant amount of computer time needed when hub shears and aeroelastic stability constraints are calculated, it is important to use approximations to the constraints and objective function. Sensitivity analysis and analytical derivatives play an important role since they can provide large reductions of computer time compared to generating such derivatives by finite differences.[29-32] Use of intermediate design variables such as cross-sectional stiffnesses also increase the efficiency of the optimization[29,30,55] process; however, it is important to provide mathematical relations that connect these variables to the actual physical dimensions that characterize the complicated multicell construction used in practical rotor blades.

An excellent "white paper" that presents an assessment of the current state of integrated multidisciplinary optimization of rotorcraft including an intelligent plan for future development is contained in a comprehensive report edited by Adelman and Mantay.[57] This ambitious and systematic plan strives toward the complete integration of the disciplines of aerodynamics, dynamics, structures, and acoustics and envisions the treatment of the complete coupled rotor/fuselage system. It is evident from this document that rotorcraft is one of the areas where an integrated multidisciplinary design approach offers excellent potential for performance gains.

In addition to the areas mentioned in Ref. 57, active control for vibration reduction and blade stability augmentation,[58] together with treatment of handling qualities criteria,[36] should also be included in the scope of integrated multidisciplinary optimization.

It should also be mentioned that the tools are available to carry out fuselage optimization studies for minimum weight design with low-vibration level type of constraints.[45] However, the details of coupling the rotor into the fuselage so as to have a good analytical model for the propagation of the vibrations from the rotor into the airframe requires considerable additional research. Thus, the coupled rotor/fuselage vibration reduction problem presents a considerable challenge for the future.

Finally, it should be noted that structural optimization for vibration reduction in helicopters is not an alternative to the use of active controls such as higher harmonic control (HHC)[58] for vibration reduction. It is only by combining modern structural optimization with HHC that the next generation of vibration-free rotorcraft can be designed.

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References

[1]Reichert, G., "Helicopter Vibration Control – A Survey," *Vertica,* Vol. 5, No. 1, 1981, pp. 1-20.

[2]Loewy, R. G., "Helicopter Vibrations: A Technological Perspective," *Journal of the American Helicopter Society,* Vol. 29, No. 4, 1984, pp. 4-30.

[3]Miller, R. H., and Ellis, C. W., "Helicopter Blade Vibration and Flutter," *Journal of the American Helicopter Society*, Vol. 1, No. 3, July 1956, pp. 19-38.

[4]Hirsch, H., Dutton, R. E., and Rasamoff, A., "Effects of Spanwise and Chordwise Mass Distribution on Rotor Blade Cyclic Stresses," *Journal of the American Helicopter Society*, Vol. 1, No. 2, April 1956, pp. 37-45.

[5]McCarthy, J. L., and Brooks, G. W., "A Dynamic Model Study of the Effect of Added Weights and Other Structural Variations on the Blade Bending Strains of a Experimental Two-Blade Jet-Driven Helicopter in Hovering and Forward Flight," NACA TN-3367, May 1955.

[6]Daughaday, H., DuWaldt, F., and Gates, C., "Investigation of Helicopter Blade Flutter and Load Amplication Problem," *Journal of the American Helicopter Society,* Vol. 2, No. 3, July 1957, pp. 27-45.

[7]Blackwell, R. H., "Blade Design for Reduced Helicopter Vibration," *Journal of the American Helicopter Society*, Vol. 28, No. 3, July 1983, pp. 33-41.

[8]Schmit, L. A., "Structural Synthesis from Abstract Concept to Practical Tool," AIAA Paper 77-395, March 1977.

[9]Schmit, L. A., "Structural Synthesis—its Genesis and Development," *AIAA Journal*, Vol. 18, No. 10, 1981, pp. 1249-1263.

[10]Schmit, L. A., "Structural Optimization—Some Key Ideas and Insights," *New Directions in Optimum Structural Design,* edited by E. Atrek, R. H. Gallagher, K. M. Radsdell, and O. C. Zienkiewicz, John Wiley and Sons, 1984.

[11]Schmit, L. A., and Miura, H., "Approximation Concepts for Efficient Structural Analysis," NASA CR-2552, 1976.

[12]Ashley, H., "On Making Things the Best—The Aeronautical Uses of Optimization," *Journal of Aircraft,* Vol. 19, No. 1, 1982, pp. 5-28.

[13]Haftka, R. T., "Structural Optimization with Aeroelastic Constraints—A Survey of U.S. Applications," International Symposium on Aeroelasticity, Nuremberg, Federal Republic of Germany , Oct. 1981, DGLR-Benricht 82-01, pp. 179-186.

[14]Miura, H., "Applications of Numerical Optimization Methods to Helicopter Design Problems—A Survey," *Vertica,* Vol. 9, No. 2, 1985, pp. 141-154.

[15]Friedmann, P. P., "Application of Modern Structural Optimization to Vibration Reduction in Rotorcraft," *Vertica*, Vol. 9, No. 4, 1985, pp. 353-376.

[16]Friedmann, P. P., and Shanthakumaran, P., "Aeroelastic Tailoring of Rotor Blades for Vibration Reduction in Forward Flight," AIAA Paper 83-0916, April 1983.

[17]Friedmann, P. P., and Shanthakumaran, P., "Optimum Design of Rotor Blades for Vibration Reduction in Forward Flight," *Journal of the American Helicopter Society*, Vol. 29, No. 4, Oct. 1984, pp. 70-80.

[18]Shanthakumaran, P., "Optimum Design of Rotor Blades for Vibration Reduction in Forward Flight," Ph.D. Dissertation, Mechanics and Structures Dept. Univ. of California, Los Angeles, 1982.

[19]Friedmann, P. P., and Straub, F. K., "Application of the Finite Element Method to Rotary-Wing Aeroelasticity," *Journal of the American Helicopter Society*, Vol. 25, No. 1, Jan. 1980, pp. 36-44.

[20]Friedmann, P., and Kottapalli, S. B. R., "Coupled Flap-Lag Torsional Dyanmics of Hingeless Rotor Blades in Forward Flight," *Journal of the American Helicopter Society*, Vol. 27, No. 4, Oct. 1982, pp. 28-36.

[21]Miura, H., and Schmit, L. A., "NEWSUMT—A Fortran Program for Inequality Constrained Function Minimization—User's Guide," NASA CR-159070, 1979.

[22]Haftka, R. T., and Kamat, M. P., *Elements of Structural Optimization*, Martinus Nijhoff Publishers, 1985, Chap. 5.

[23]Miura, H., and Schmit, L. A., "Second Order Approximation of Natural Frequency Constraints in Structural Synthesis," *International Journal of Numerical Methods in Engineering*, Vol. 13, No. 2, 1978, pp. 337-351.

[24]Celi, R., and Friedmann, P. P., "Structural Optimization with Aeroelastic Constraints of Rotor Blades with Straight an Swept Tips," AIAA Paper 88-2297, April 1988; also, *AIAA Journal*, Vol. 28, No. 5, pp. 928-936.

[25]Celi, R., "Aeroelasticity and Structural Optimization of Helicopter Rotor Blades with Swept Tips," Ph.D. Dissertation, Mechanical, Aerospace an Nuclear Engineering Dept., Univ. of California, Los Angeles, Oct. 1987.

[26]Vanderplaats, G. N., "Approximation Concepts for Numerical Airfoil Optimization," NASA TP-1370, March 1979.

[27]Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design with Applications*, McGraw Hill Book Co., New York, 1984, pp. 211-215.

[28]Vanderplaats, G. N., "CONMIN—A Fortran Program for constrained Function Minimization; User's Manual," NASA TM-X-62,282, Aug. 1973.

[29]Celi, R., and Friedmann, P. P., "Rotor Blade Aeroelasticity in Forward Flight with an Implicit Aerodynamic Formulation," *AIAA Journal*, Vol. 26, No. 12, 1988, pp. 1425-1433.

[30]Celi, R., and Friedmann, P. P., "Aeroelastic Modeling of Swept Tip Rotor Blades Using Finite Elements," *Journal of the American Helicopter Society,* Vol. 33, No. 2, April 1988, pp. 43-52.

[31]Lim, J., and Chopra, I., "Aeroelastic Optimization of a Helicopter Rotor," *Proceedings of 44th Annual Forum of the American Helicopter Society,* Washington DC, June 1988, pp. 545-558.

[32]Lim, J. W., "Aeroelastic Optimization of a Helicopter Rotor," Ph.D. Disseration, Dept. of Aerospace Engineering, Univ. of Maryland, College Park, MD, 1988.

[33]Lim, J., and Chopra, I., "Design Sensitivity Analysis for an Aeroelastic Optimization of a Helicopter Rotor," AIAA Paper 87-0923, April 1987.

[34]Lim, J., and Chopra, I., "Stability Sensitivity Analysis for the Aeroelastic Optimization of a Helicopter Rotor," AIAA Paper 88-2310, April 1988.

[35]Adelman, H. M., and Haftka, R. T., "Sensitivity Analysis of Discrete Structural Systems," *AIAA Journal*, Vol. 24, No. 5, 1986, pp. 823-832.

[36]Peters, D. A., Rossow, M. P., Korn, A., and Ko, T., "Design of Helicopter Rotor Blades for Optimum Dynamic Characteristics," *Computers and Mathematics with Applications*, Vol. 12A, No. 1, Jan., 1986, pp. 85-109.

[37]Peters, D. A., and Cheng, Y. P., "Optimization of Rotor Blades for Combined Structural Performance and Aeroelastic Constraints," *Proceedings of a Symposium on Recent Advances in Multidisciplinary Analysis and Optimization*, NASA CP-3031, Pt. 1, Sept. 1988, pp. 163-180.

[38]Celi, R., "Optimum Design of Helicopter Rotors for Longitudinal Handling Qualities Improvement in Forward Flight," AIAA Paper 89-1270, April 1989.

[39]Bielawa, R. L., "Techniques for Stability Analysis and Design Optimization with Dynamic Constrains of Nonconservative Linear Systems," AIAA Paper 71-388, 1971.

[40]Taylor, R. B., "Helicopter Vibration by Rotor Blade Modal Shaping," *Proceedings of the 38th Annual Forum of the American Helicopter Society*, Anaheim, CA, May, pp. 90-101.

[41]Davis, M. W., and Weller, W. H., "Application of Design Optimization Techniques to Rotor Dynamics Problems," *Journal of the American Helicopter Society*, Vol. 33, No. 3, July 1988, pp. 42-50.

[42]Weller, W. H., and Davis, M. W., "Experimental Verification of Helicopter Blade Designs Optimized for Minimum Vibration," *Proceedings of the 44th Annual National Forum of the American Helicopter Society*, Washington, DC, June 1988, pp. 263-279.

[43]Weller, W. H., and Davis, M. W., "Wind Tunnel Tests of Helicopter Blade Designs Optimized for Minimum Vibrations," *Journal of the American Helicopter Society*, Vol. 34, No. 3, July 1989, pp. 40-50.

[44]Vanderplaats, G. N., Sugimoto, H., and Sprague, C. M., "ADS-1: A New General Purpose Optimization Program," *AIAA Journal*, Vol. 22, No. 10, 1984, pp. 1458-1459.

[45]Bennett, R. L., "Application of Optimization Methods to Rotor Design Problems," *Vertica,* Vol. 7, No. 3, 1983, pp. 201-208.

[46]Yen, J. G., "Coupled Aeroelastic Hub Loads Reduction," American Helicopter Society/Nanjing Aeronautical Institute International Seminar, Nanjing, China, Nov. 1985.

[47]Banerjee, D., and Shanthakumaran, P., "Application of Numerical Optimization Methods in Helicopter Industry," *Vertica,* Vol. 13, No. 1, 1989, pp. 17-42.

[48]Hamilton, B., Peters, J., and Callahan, C., "Optimal Design of an Advanced Composite Rotating Flexbeam," *Proceedings 44th Annual American Helicopter Society Forum*, Washington, DC, June 1988, pp. 409-421.

[49]Gerstenberger, W., and Wood, E. R., "Analysis of Helicopter Aeroelastic Characteristics in High-Speed Flight," *AIAA Journal,* Vol. 1, No. 10, 1963, pp. 2366-2381.

[50]Wood, E. R., et al., "Implementation and Verification of a Comprehensive Helicopter Coupled Rotor-Fuselage Analysis," America Helicopter Society/Nanjing Aeronautical Inst. Specialist Meeting, Nanjing, China, Nov. 1985.

[51] McIntosh, S. C., "A Feasibility Study of Bearingless Rotor Flex Beam Optimization," TR-243, Nielsen Engineering and Research, Inc., Mountain View, CA, 1983.

[52]Perley, R., "Regression Analysis as a Design Optimization Tool," *Proceedings of Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization*, NASA CP-2327, Pt. 2, April 1984, pp. 579-593.

[53]Pritchard, J. L., and Adelman, H. M., "Optimal Placement of Tuning Masses for Vibration Reduction in Helicopter Rotor Blades," AIAA Paper 88-2312, April 1988.

[54]Chattopadhyay, A., and Walsh, J. L., "Minimum Weight Design of Rotorcraft Blades with Multiple Frequency and Stress Constraints," AIAA Paper 88-2337, April 1988.

[55]Chattopadhyay, A., Walsh, J. L., and Riley, M., "Integrated Aerodynamic/Dynamic Optimization of Helicopter Rotor Blades," AIAA Paper 89-1269, April 1989.

[56]Johnson, W., "A Comprehensive Analytical Model of Rotorcraft Aerodynamics and Dynamics," Part II: User's Manual, NASA TM-81183, June 1980.

[57]Adelman, H. M., and Mantay, W. R., (eds.), "Integrated Multidisciplinary Optimization of Rotorcraft: A Plan for Development," NASA TM-101617, May 1989.

[58]Friedmann, P. P., "Helicopter Rotor Dynamics and Aeroelasticity: Some Key Ideas and Insight," *Procedings of the Fifteenth European Rotorcraft Forum*, Paper No. 49, Sept. 1989, Amsterdam, Holland; also, *Vertica*, Vol. 14, No. 1, pp. 101-121.